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Basics Concepts of Signal and Systems, Detection Theory

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What is a signal?

- A signal is a set of data.

- A signal is the dependent variable or function of one or more independent variables that carries some information to represent a physical phenomenon.
  - Any time varying physical quantity
    - Independent variable: time (t), space (x, x=[x1,x2], x=[x1,x2,x3])
    - Electrocardiography signal (EEG) 1D, voice 1D, music 1D
    - Images (2D), video sequences (2D+time), volumetric data (3D)

Type of Signal

- Single Variable Signal

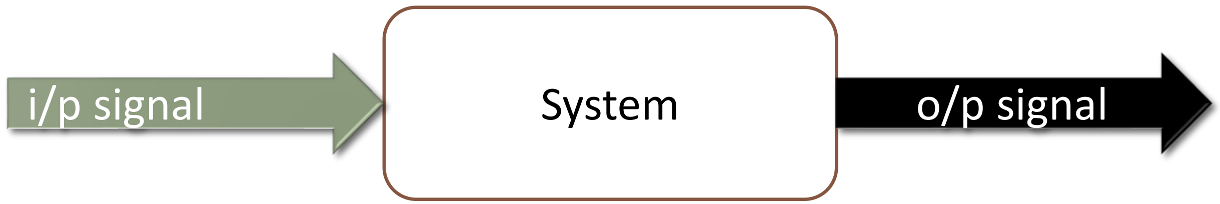
  $f(x)$, $g(t)$

- Multi Variable Signal

  $f(x_1, x_2)$, $g(t_1, t_2)$

System

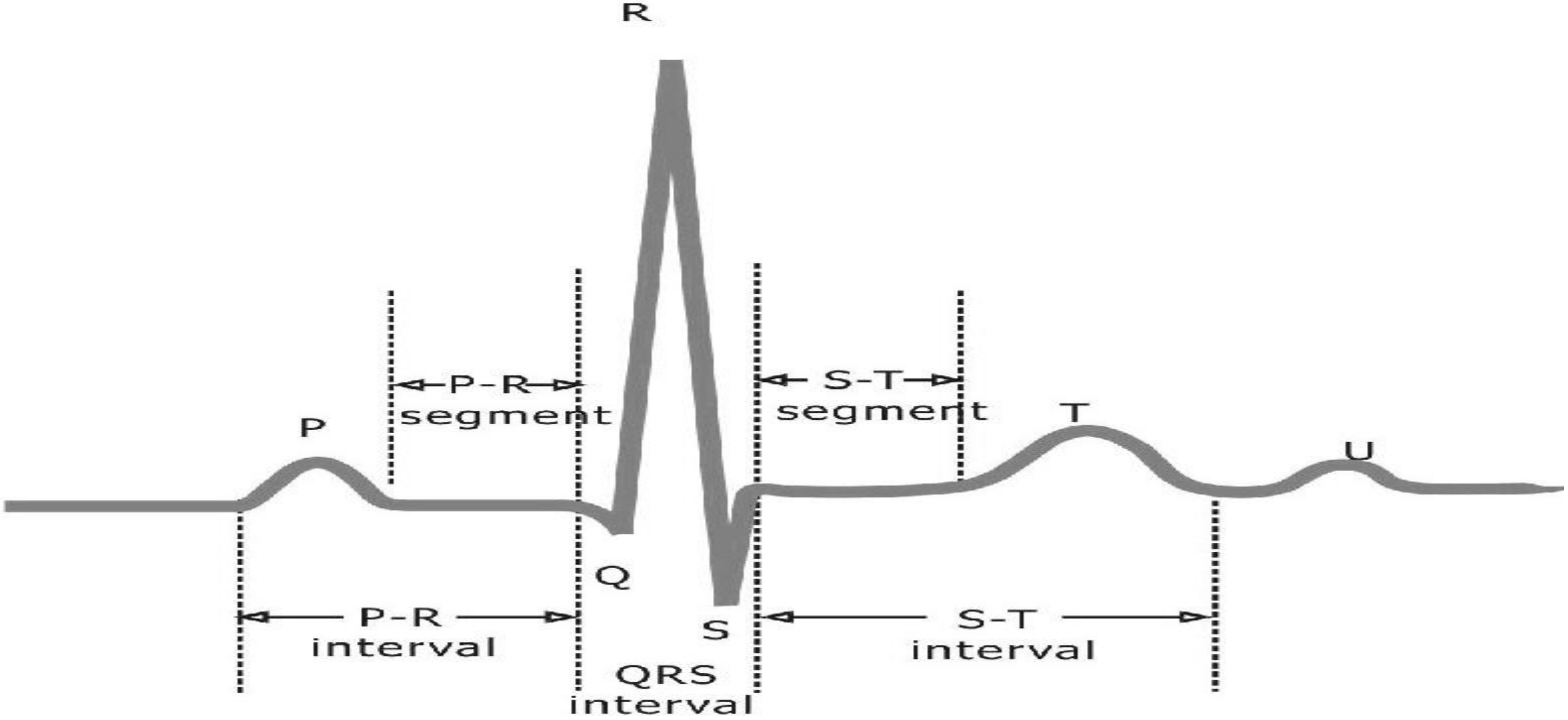
- The meaningful interconnection of physical devices and components is known as systems.



i/p signal- $F(x_1, x_2)$

o/p signal- $G(x_1, x_2)$

Example: 1D biological signals: ECG



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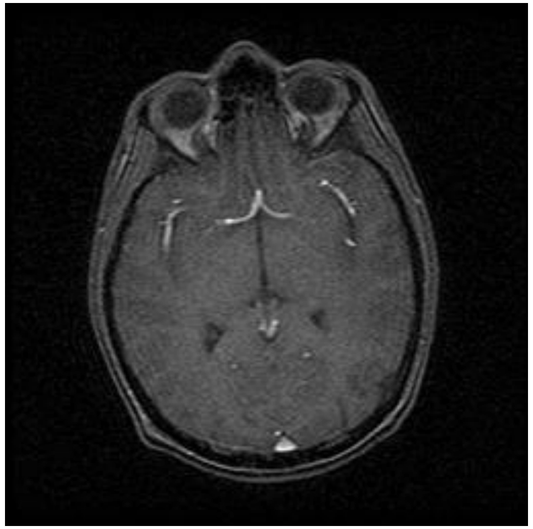
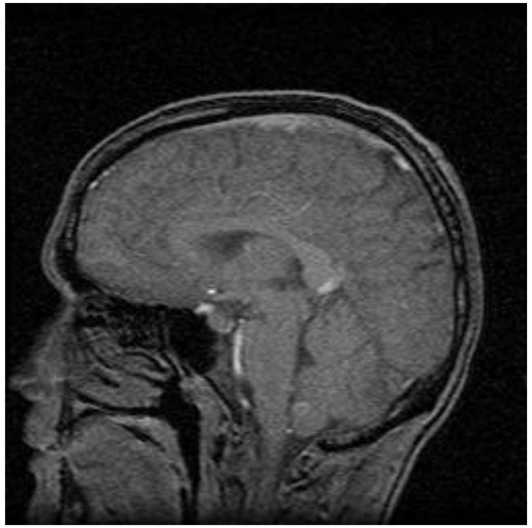
Example: 1D biological signals: EEG

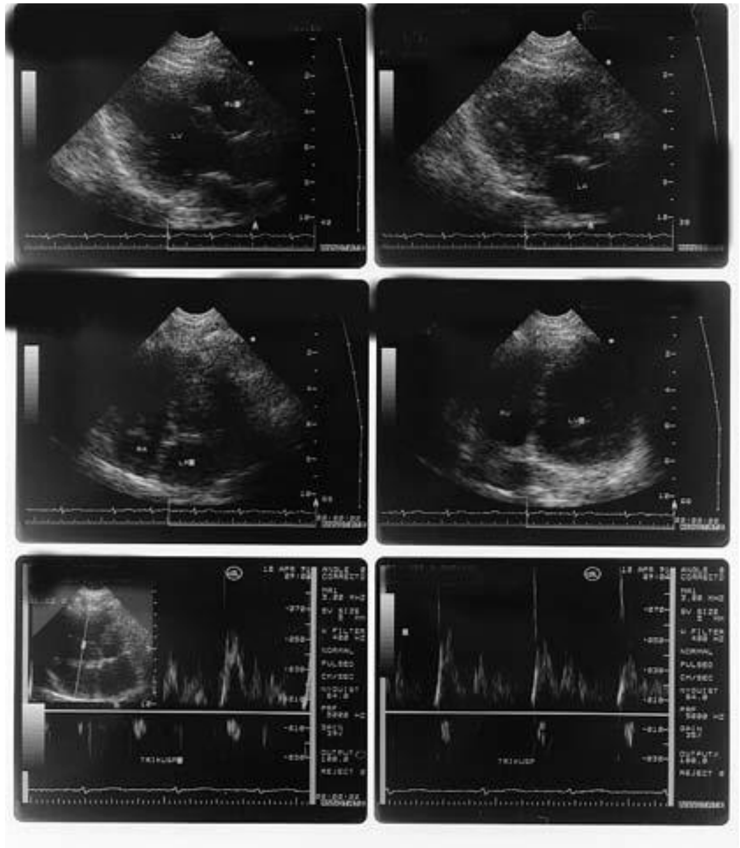
Example: 2D biological signals: MI

US

MRI

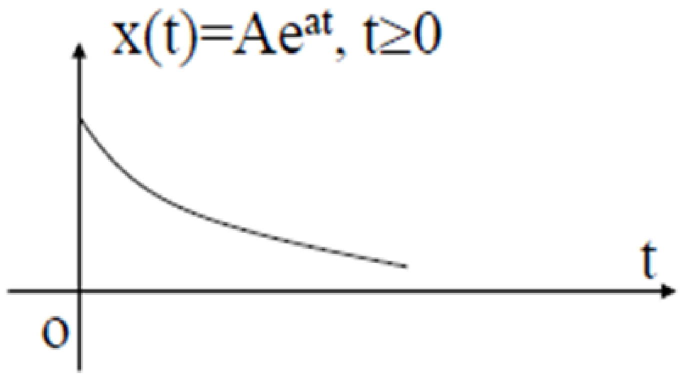




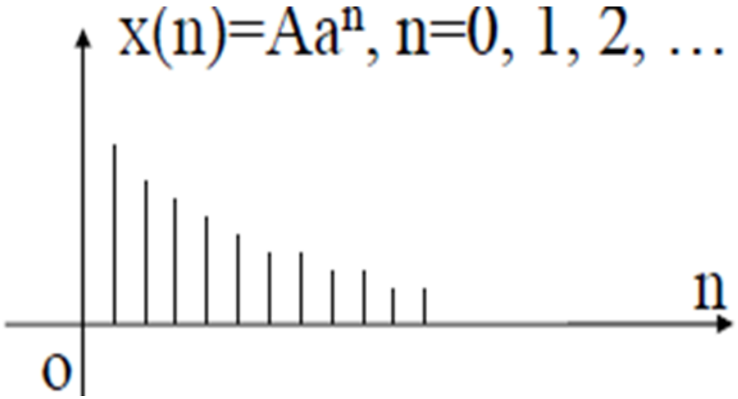


Continuous and Discrete Time Signals

- Continuous Time Signals
  - Specified for every value of time(t)



- Discrete Time Signals
  - Specified at discrete time intervals



Elementary Signals

Sinusoidal & Exponential Signals

- Sinusoids and exponentials are important in signal and system analysis.
- Sinusoidal Signals can expressed in either of two ways :
  - cyclic frequency *form*- $A \sin 2\Pi f_o t = A \sin(2\Pi/T)$
  - radian frequency form- $A \sin \omega_o t$

where $\omega_o = 2\Pi f_o = 2\Pi/T_o$

T_o = Time Period of the Sinusoidal Wave

Elementary Signals

$x(t) = A \sin(2\Pi f_o t + \theta)$
$= A \sin(\omega_o t + \theta)$

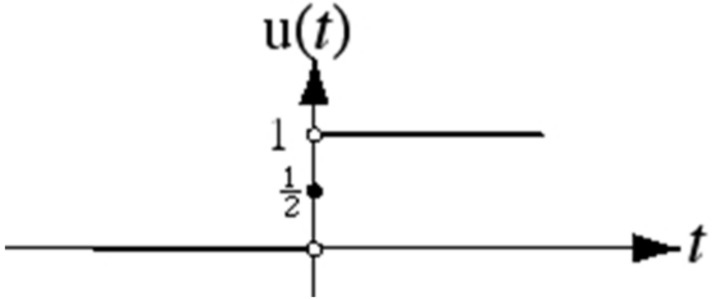
$x(t) = Ae^{at}$ Real Exponential
$= Ae^{j\omega t} = A[\cos(\omega_o t) + j \sin(\omega_o t)]$ Complex Exponential

θ = Phase of sinusoidal wave
A = amplitude of a sinusoidal or exponential signal
f_o = fundamental cyclic frequency of sinusoidal signal
ω_o = radian frequency

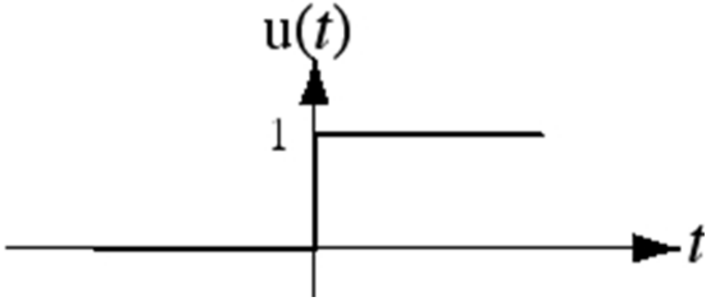
Unit Step Function

$$\mathrm{u}(t) = \begin{cases} 1 & , \quad t > 0 \\ 1/2 & , \quad t = 0 \\ 0 & , \quad t < 0 \end{cases}$$

Precise Graph



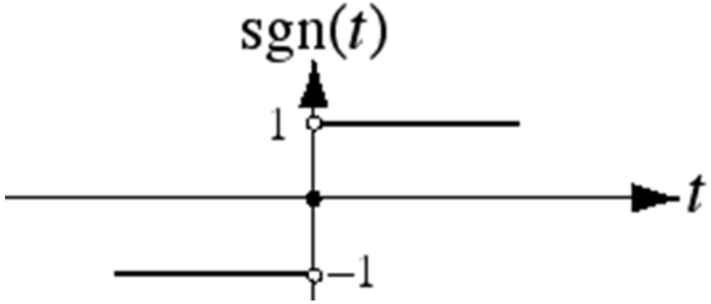
Commonly-Used Graph



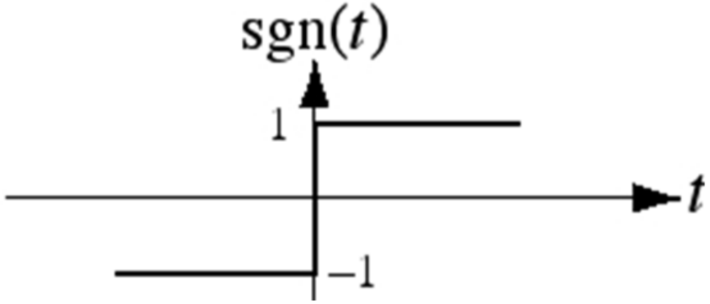
Signum Function

$$\operatorname{sgn}(t) = \begin{cases} 1 & , \; t > 0 \\ 0 & , \; t = 0 \\ -1 & , \; t < 0 \end{cases} \; = \; 2\mathrm{u}(t) - 1$$

Precise Graph



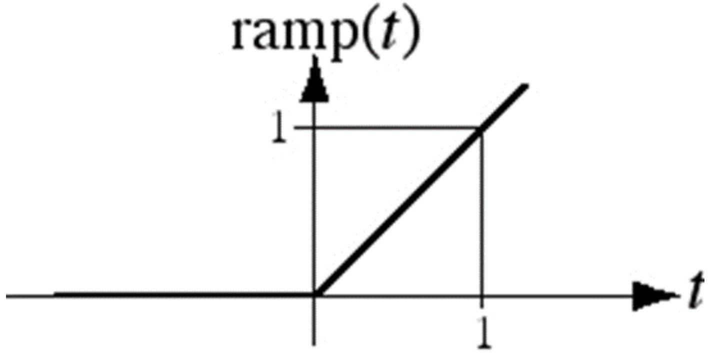
Commonly-Used Graph



The signum function, is closely related to the unit-step function.

Unit Ramp Function

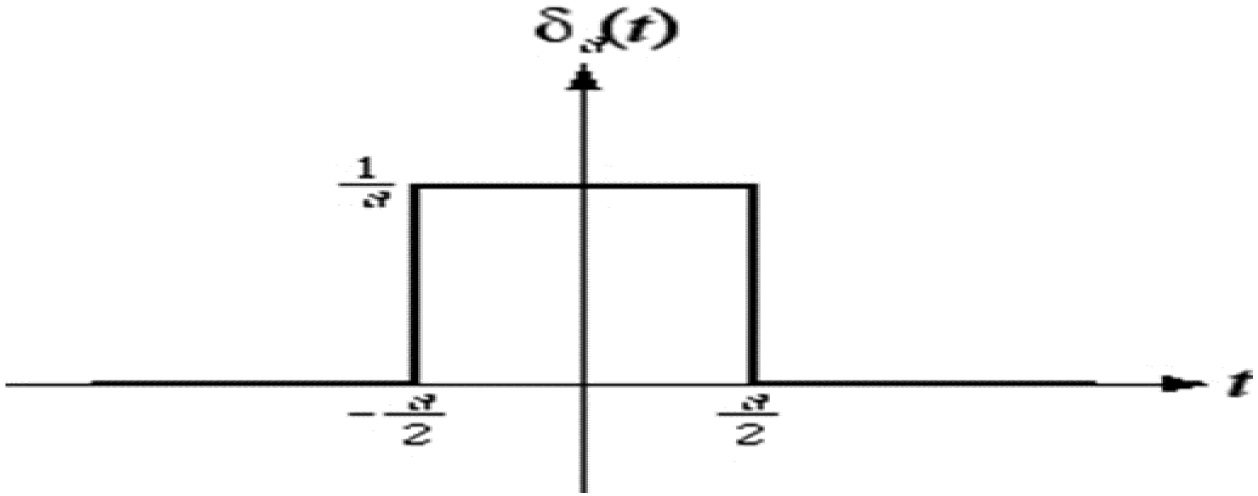
$$\text{ramp}(t)=\begin{cases} t, t > 0 \\ 0, t \leq 0 \end{cases}$$



- It is called the unit ramp function because for positive t, its slope is one amplitude unit per time.

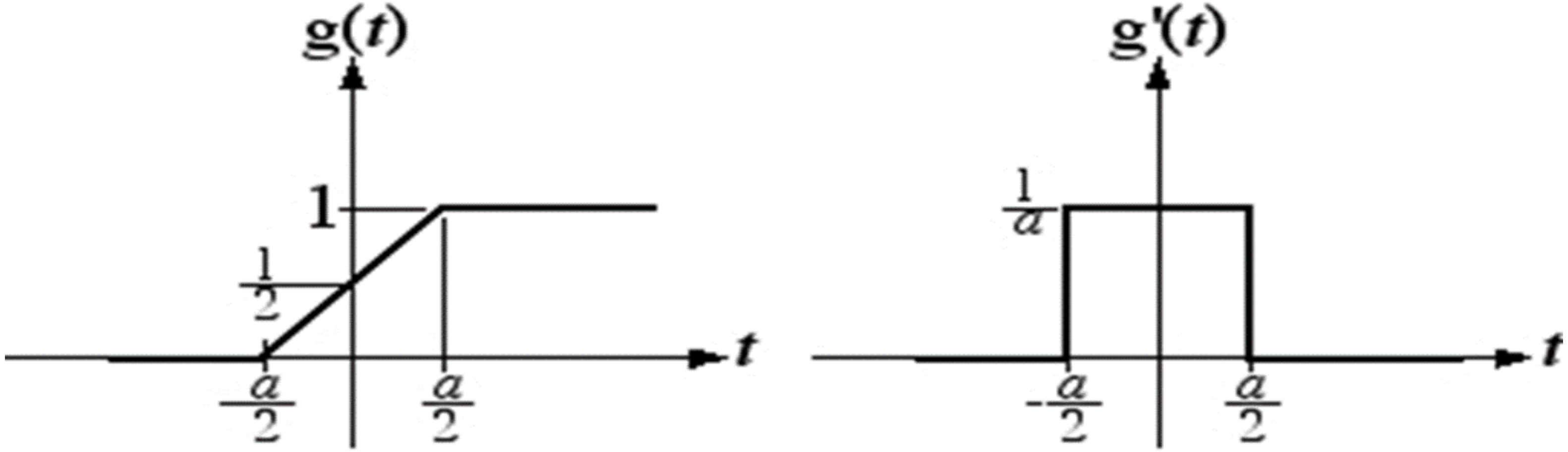
Rectangular Pulse or Gate Function

$$\delta_a(t) = \begin{cases} 1/a & , \quad |t| < a/2 \\ 0 & , \quad |t| > a/2 \end{cases}$$



Unit Impulse Function

As a approaches zero, $g(t)$ approaches a unit step and $g'(t)$ approaches a unit impulse

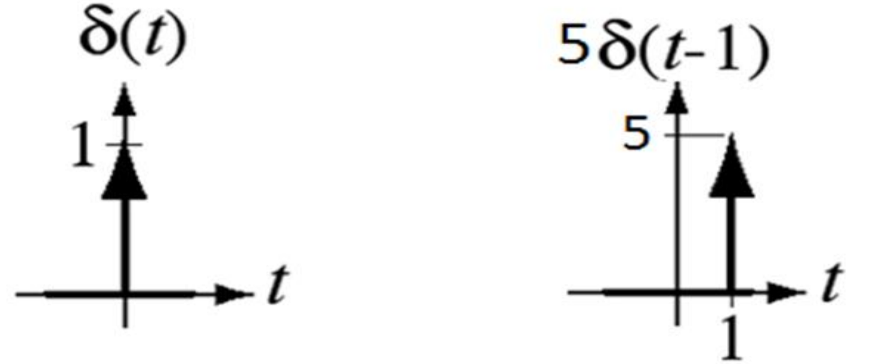


Functions that approach unit step and unit impulse

So unit impulse function is the derivative of the unit step function or unit step is the integral of the unit impulse function

Representation of Impulse Function

The area under an impulse is called its strength or weight. It is represented graphically by a vertical arrow. An impulse with a strength of one is called a unit impulse.

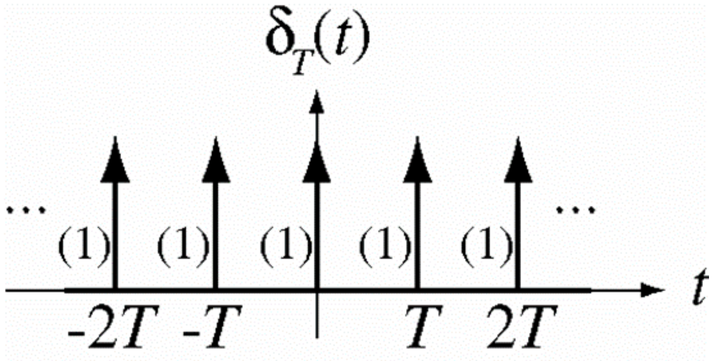


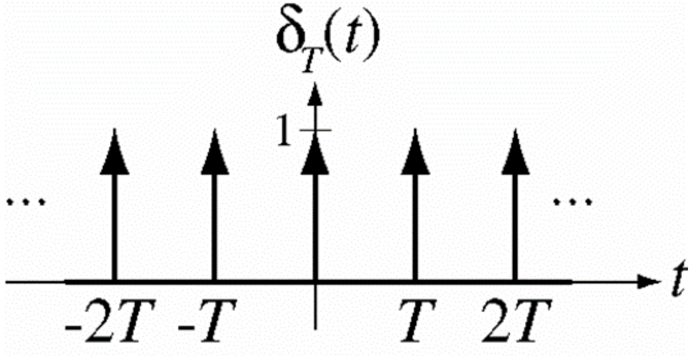
Representation of Unit Impulse   Shifted Impulse of Amplitude5

Unit Impulse Train

The unit impulse train is a sum of infinitely uniformly- spaced impulses and is given by

$$\delta_T(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT) \quad , \quad n \text{ an integer}$$

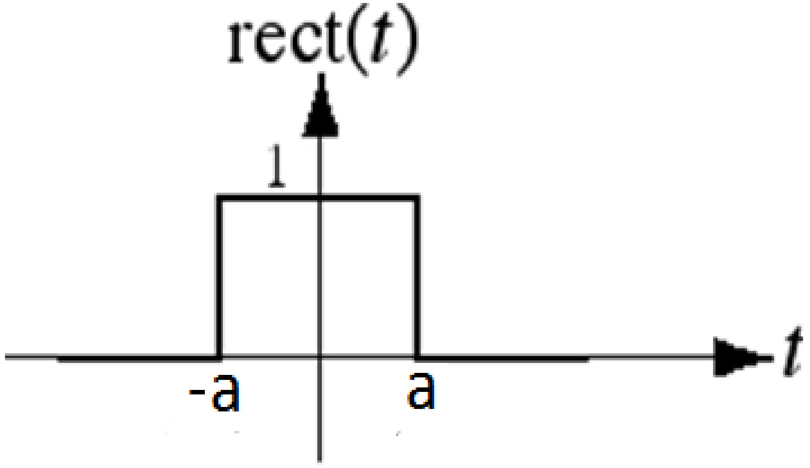




The Unit Rectangle Function

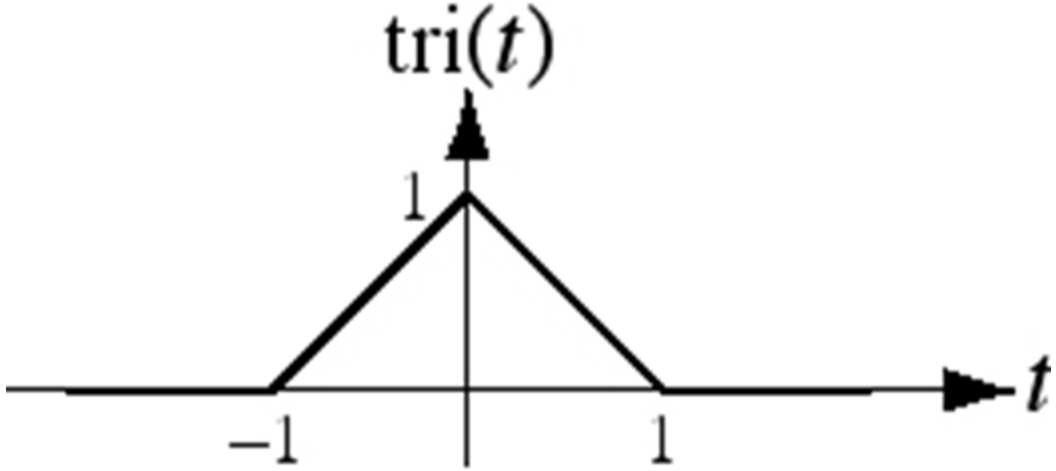
The unit rectangle or gate signal can be represented as combination of two shifted unit step signals as shown

$$\mathrm{rect}(t) = u(t+a)-u(t-a)$$



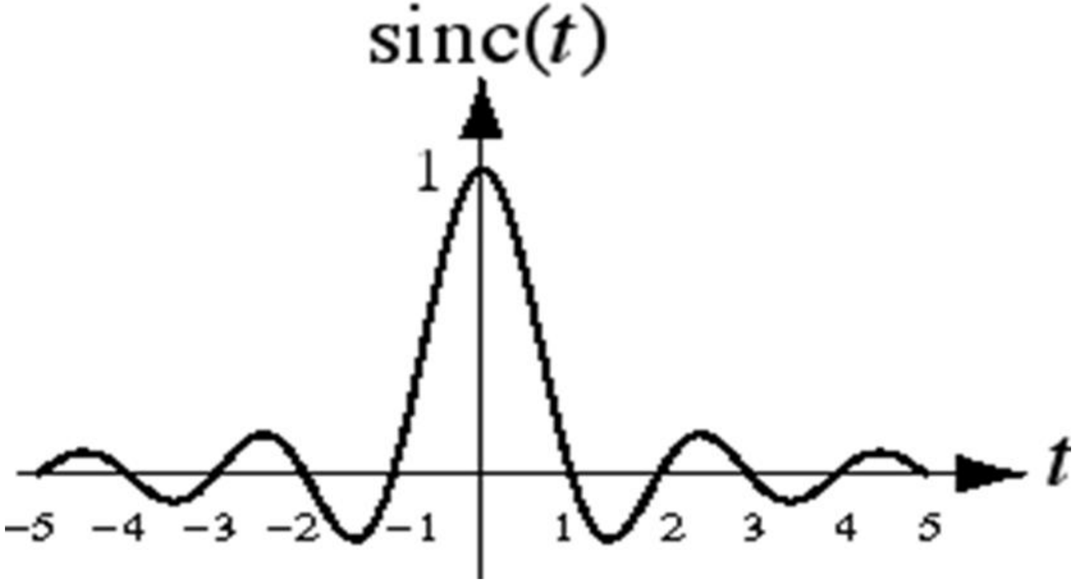
The Unit Triangle Function

$$\text{tri}(t) = \begin{cases} t+1, & -1 \leq t \leq 0 \\ -t+1, & 0 \leq t \leq 1 \end{cases}$$



Sinc Function or cardinal sinc

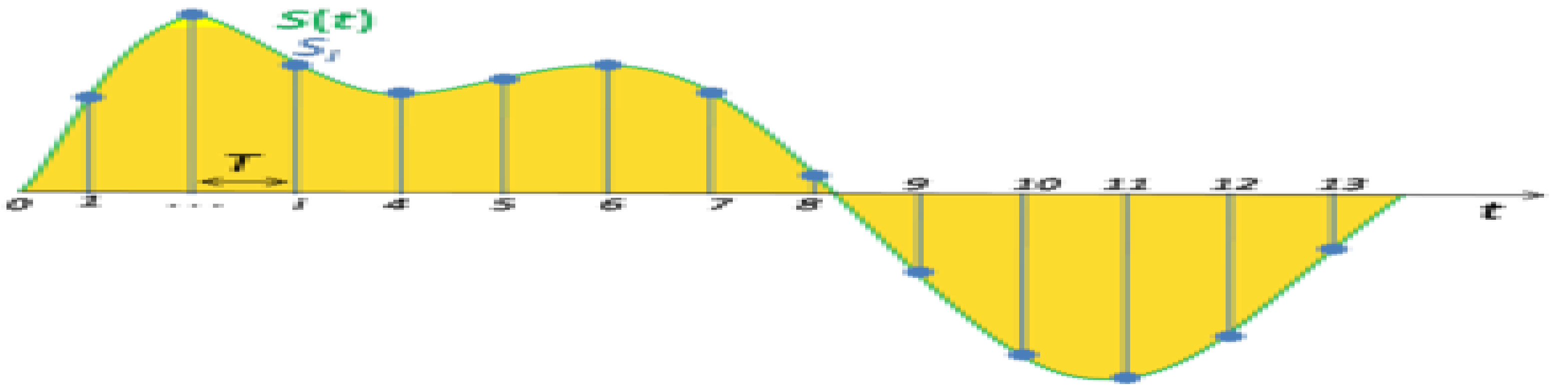
$$\mathrm{sinc}(t) = \begin{cases} 1, & t = 0 \\ \dfrac{\sin(\pi t)}{\pi t}, & t \neq 0 \end{cases}$$



Discrete-Time Signals

- **Sampling** is the reduction of continuous time signal into discrete time signal
- $x(t)$ is a continuous-time signal, $x[n]$ is a discrete-time signal

$$x[n] = x(nT_s) \text{ where } T_s \text{ is the time between samples}$$



Discrete Time Exponential and Sinusoidal Signals

DT signals can be defined in a manner analogous to their continuous-time counter part

$x[n] = A \sin(2\Pi n/N_o+\theta)$

$= A \sin(2\Pi F_o n+ \theta)$

Discrete Time Sinusoidal Signal

$x[n] = a^n$

Discrete Time Exponential Signal

n = the discrete time

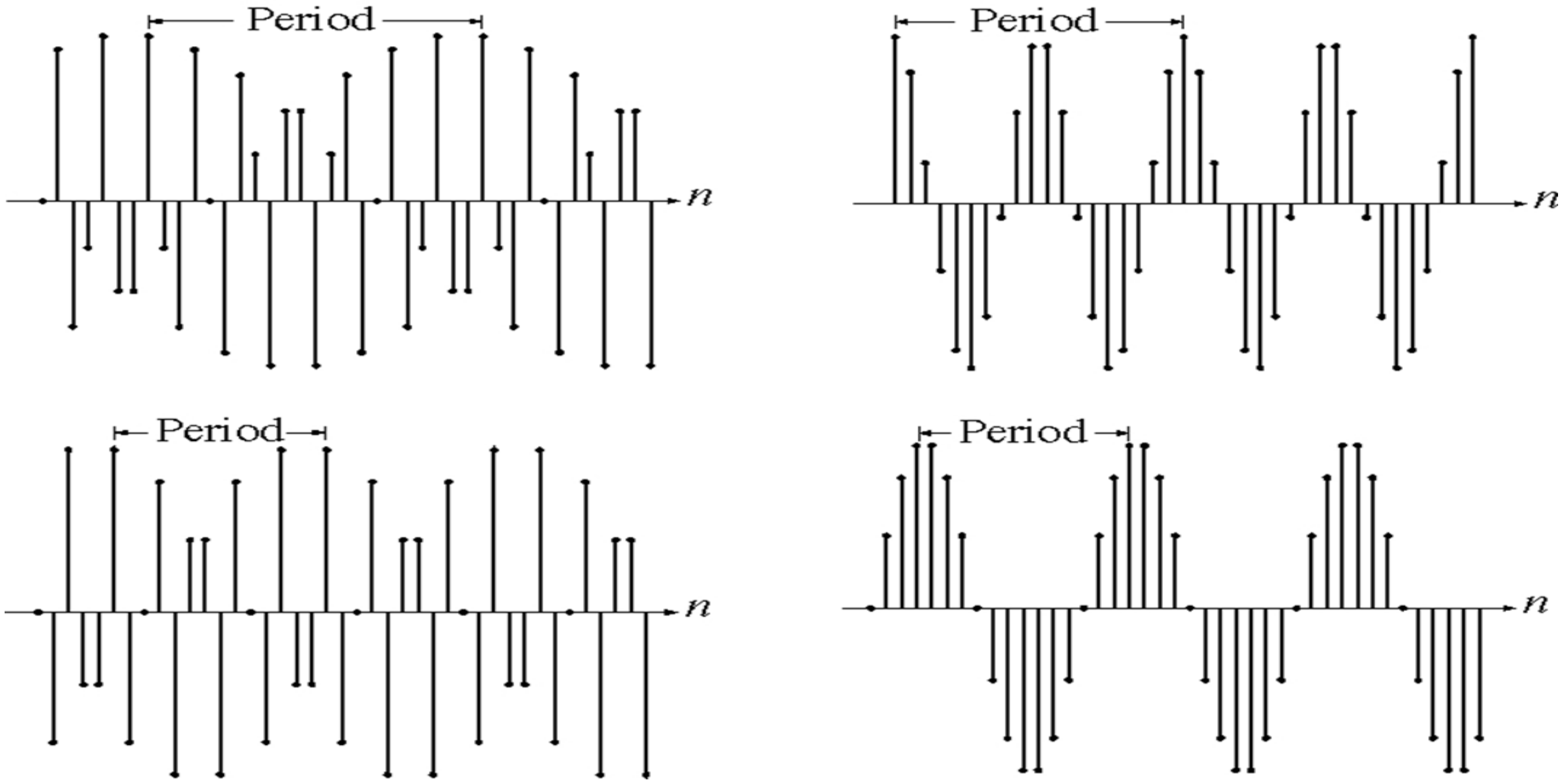
A = amplitude

θ = phase shifting radians,

N_o = Discrete Period of the wave

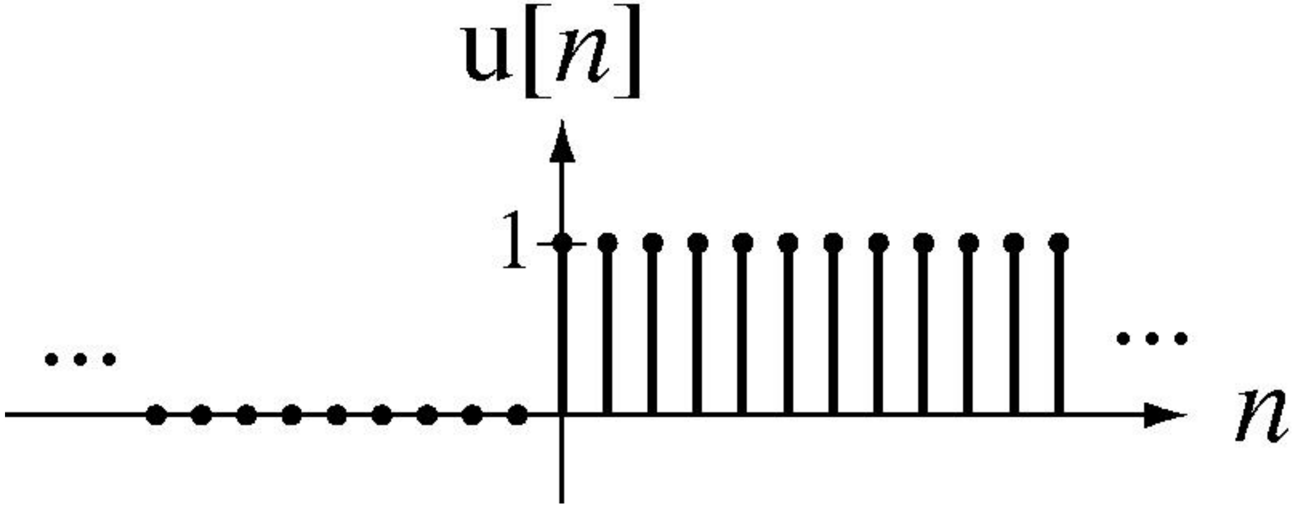
$1/N_0 = F_o = \Omega_o/2\Pi$ = Discrete Frequency

Discrete Time Sinusoidal Signals



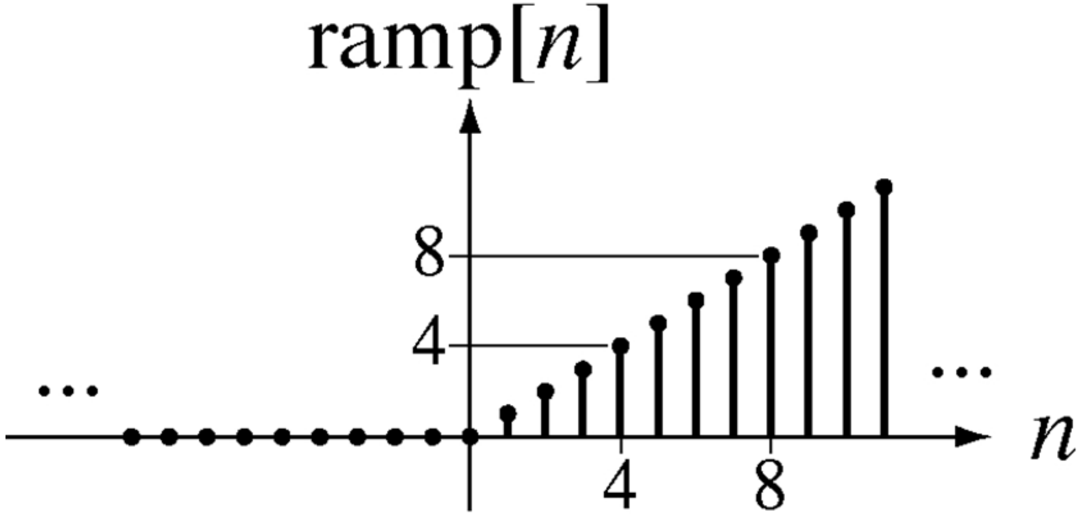
Discrete Time Unit Step Function or Unit Sequence Function

$$\mathrm{u}[n] = \begin{cases} 1 \ , & n \geq 0 \\ 0 \ , & n < 0 \end{cases}$$



Discrete Time Unit Ramp Function

$$ramp[n] = \begin{cases} n, n > 0 \\ 0, n < 0 \end{cases}$$



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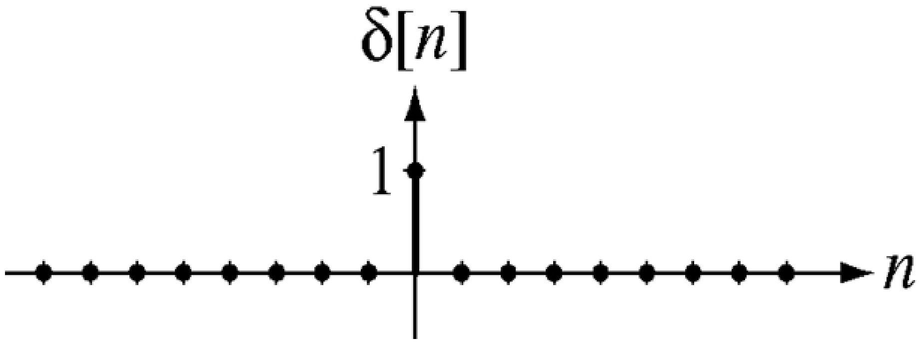
Discrete Time Unit Impulse Function or Unit Pulse Sequence

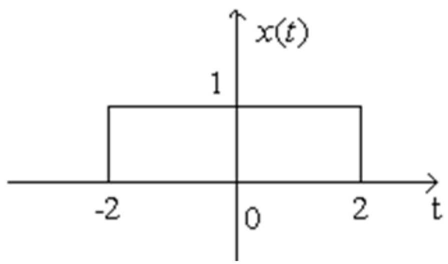
$$\delta[n] = \begin{cases} 1 \ , & n = 0 \\ 0 \ , & n \neq 0 \end{cases}$$



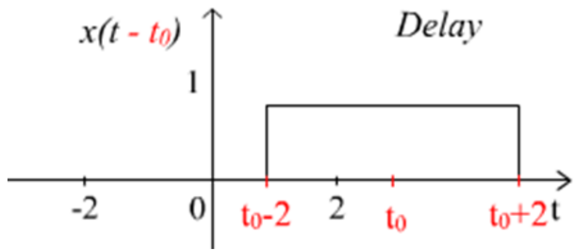
$\delta[n] = \delta[an]$ for any non-zero, finite integer a.

Time Shifting

- The original signal $x(t)$ is shifted by an amount t_o.



- X(t)→X(t-to) →Signal Delayed→ Shift to the right

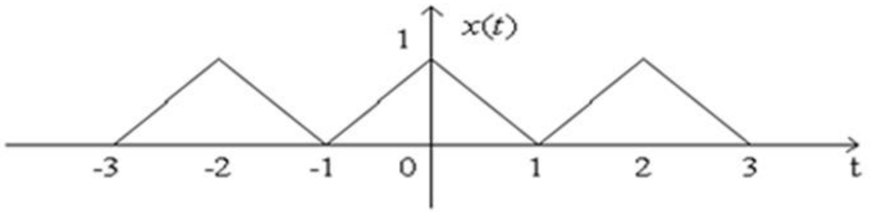


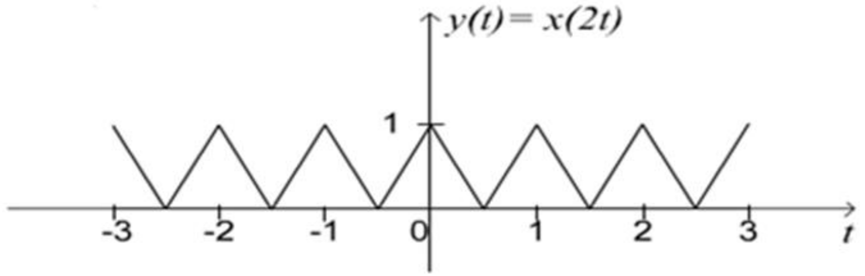
Time Scaling

- For the given function x(t), x(at) is the time scaled version of x(t)
- For a > 1,period of function x(t) reduces and function speeds up. Graph of the function shrinks.
- For a < 1, the period of the x(t) increases and the function slows down. Graph of the function expands.

Time Scaling

Example: Given $x(t)$ and we are to find $y(t) = x(2t)$.





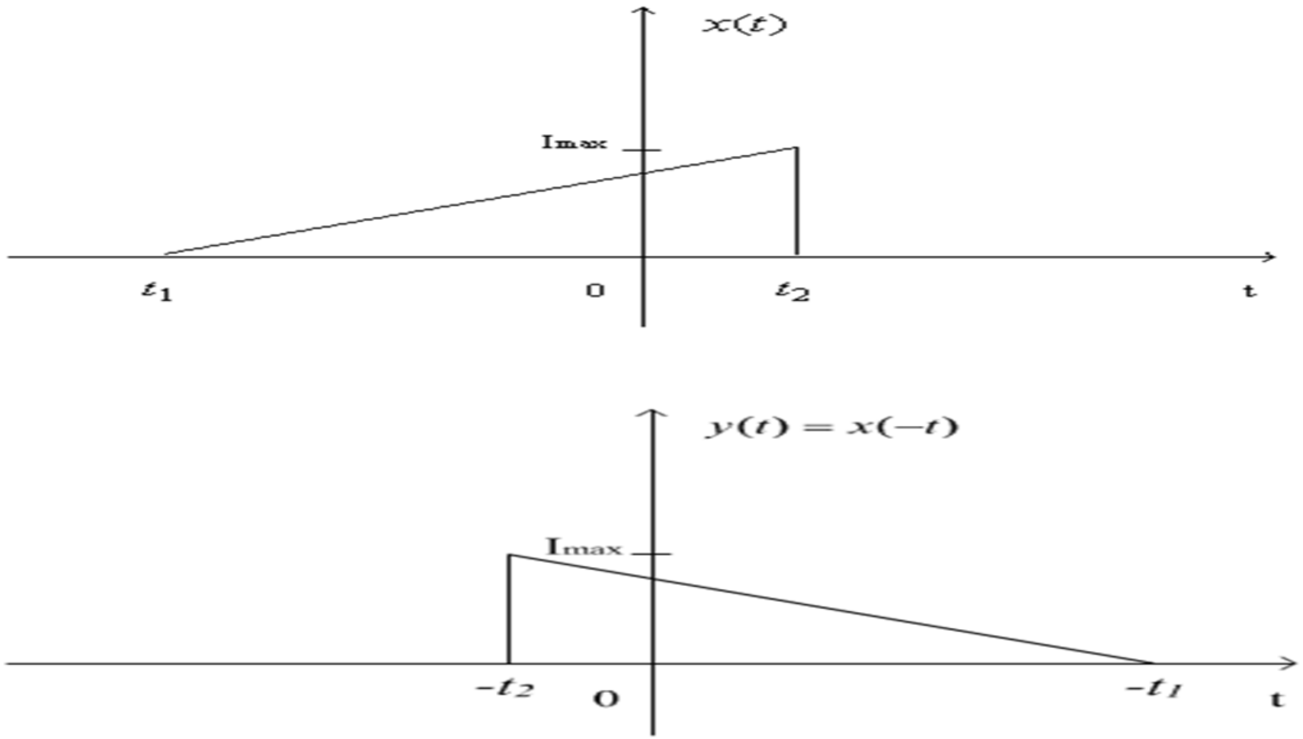
The period of $x(t)$ is 2 and the period of $y(t)$ is 1,

Time Reversal

- Time reversal is also called time folding
- In Time reversal, signal is reversed with respect to time i.e.

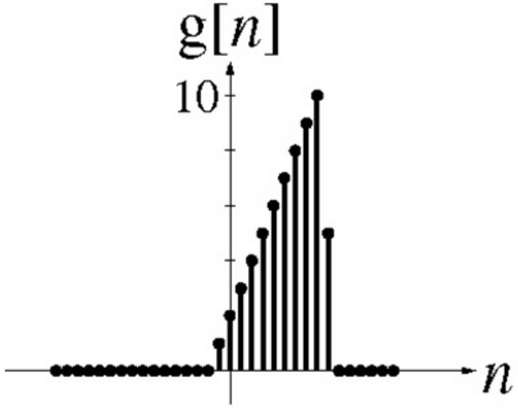
  $y(t) = x(-t)$ is obtained for the given function

Time Reversal

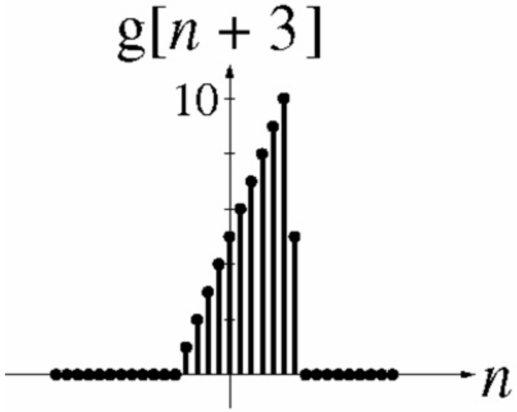


Operations of Discrete Time Functions

Time shifting $n \to n + n_0$, n_0 an integer



n	$g[n]$
–1	1
0	2
1	3
2	4
3	5
4	6
5	7
6	8
7	9
8	10
9	5
10	0



n	$n+3$	$g[n+3]$
–4	–1	1
–3	0	2
–2	1	3
–1	2	4
0	3	5
1	4	6
2	5	7
3	6	8
4	7	9
5	8	10
6	9	5
7	10	0

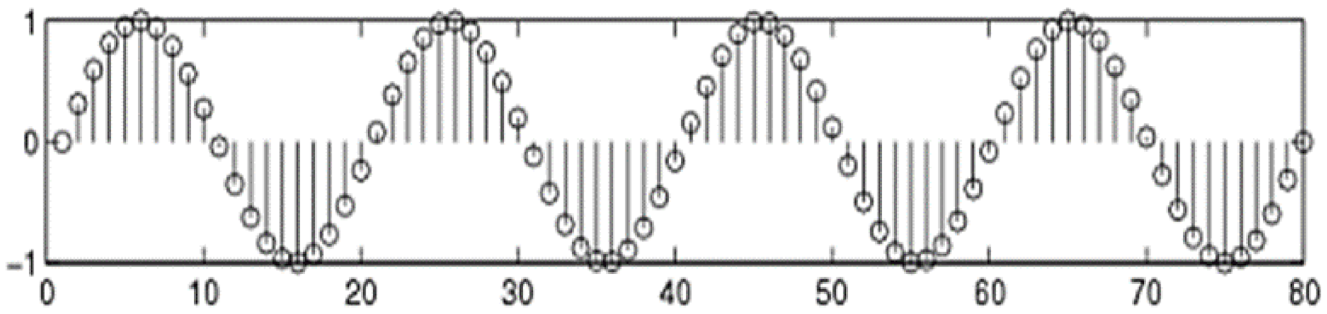
Classification of signals

- Deterministic & Non Deterministic Signals
- Periodic & Non periodic Signals
- Even & Odd Signals
- Energy & Power Signals

Deterministic & Non Deterministic Signals

Deterministic signals

- Behavior of these signals is predictable w.r.t time
- There is no uncertainty with respect to its value at any time.
- These signals can be expressed mathematically. For example $x(t) = \sin(3t)$ is deterministic signal.



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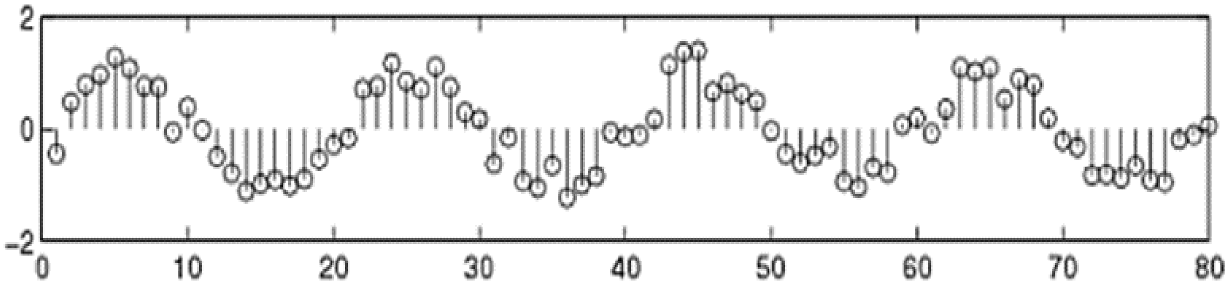
Deterministic & Non Deterministic Signals

Non Deterministic signals

- Behavior of these signals is not predictable w.r.t time
- There is an uncertainty with respect to its value at any time.
- These signals can't be expressed mathematically.
- For example Thermal Noise generated is non deterministic signal.



Periodic and Non-periodic Signals

A signal is said to be periodic if it repeats itself after regular interval of time

- Given x(t) is a continuous-time signal

x (t) is periodic iff $x(t) = x(t+nT_o)$ for any T and n and T is the smallest fixed value of time for which signal is periodic

Example

- x(t) = sin (t) at T=2 pie

- In DTS, Given x(n)=x(n+mN) is periodic for any N and must be integer
- dc value is periodic or not ?

Periodic and Non-periodic Signals

- For non-periodic signals

$$x(t) \neq x(t+nT_o)$$

- Example of non periodic signal is an exponential signal

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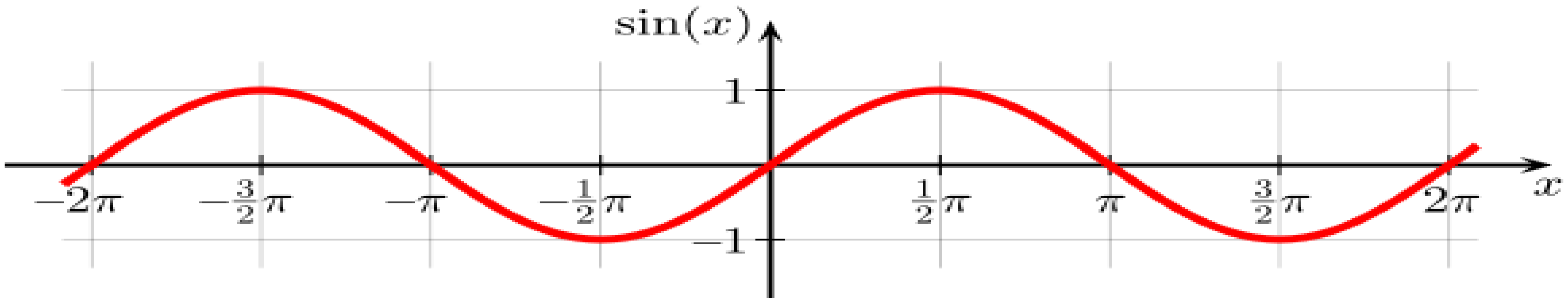
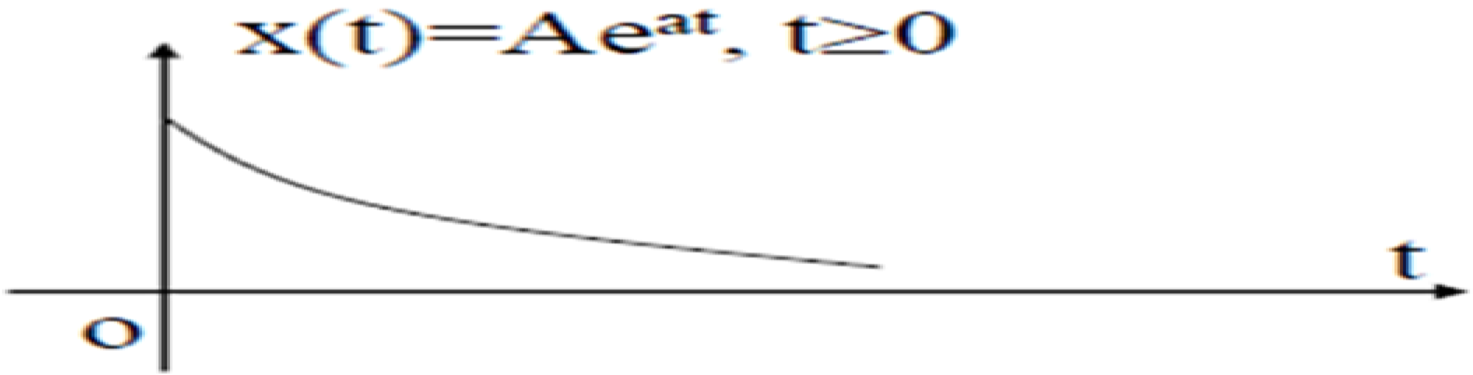
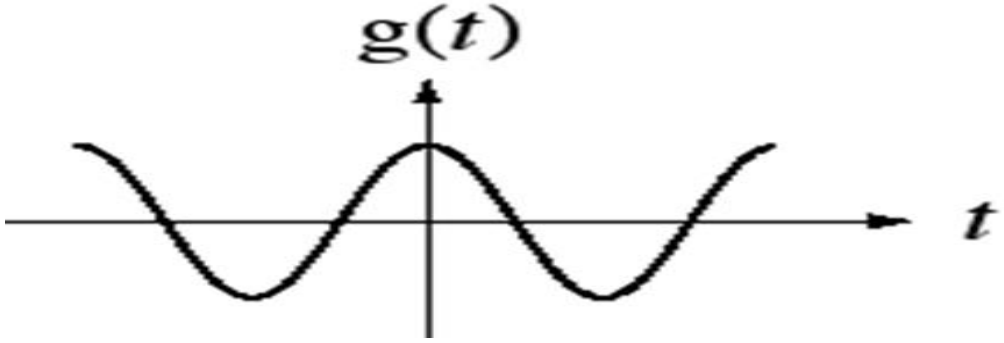
Periodic and Non-periodic Signals



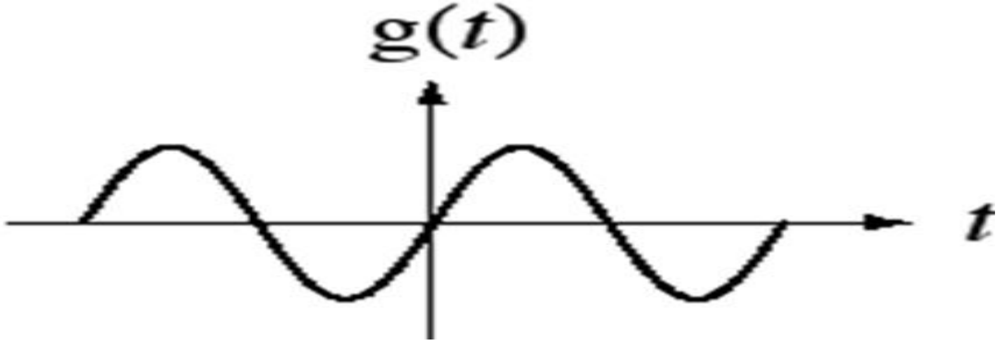


Even and Odd Signals

- remain identical under folding operation
- $x(t) \xrightarrow{T.R.} x(-t)=x(t)$ for all value of t
- Even signals can be easily spotted as they are symmetric around the vertical axis



- doesn’t remain identical under folding operation
- $x(t) \xrightarrow{T.R.} x(-t)=-(x(-t))$ for all value of t
- or $x(t) \neq x(-t)$



Decomposition of Even and Odd Functions

The **even part** of a function is $g_e(t) = \dfrac{g(t) + g(-t)}{2}$

The **odd part** of a function is $g_o(t) = \dfrac{g(t) - g(-t)}{2}$

A function whose even part is zero, is odd and a function whose odd part is zero, is even.

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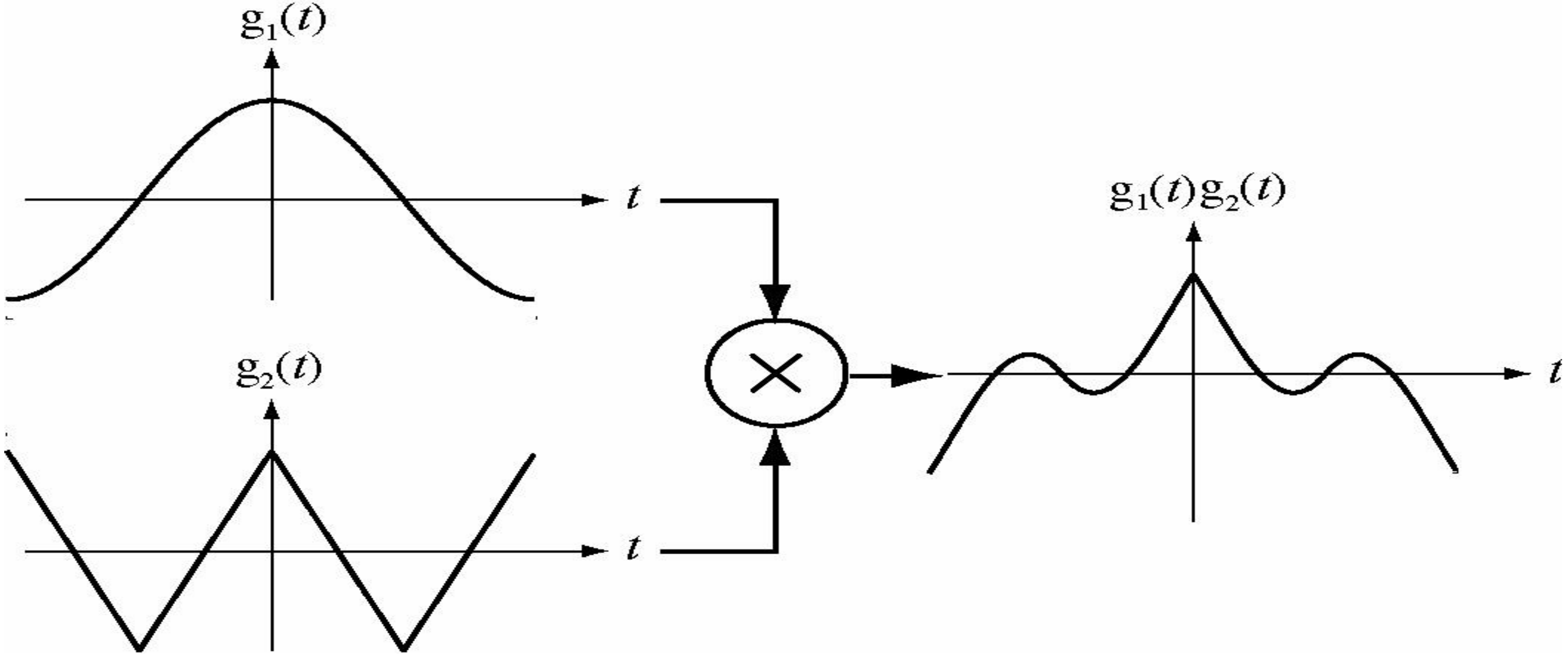
Various Combinations of even and odd functions

Function type	Sum	Difference	Product	Quotient
Both even	Even	Even	Even	Even
Both odd	Odd	Odd	Even	Even
Even and odd	Neither	Neither	Odd	Odd

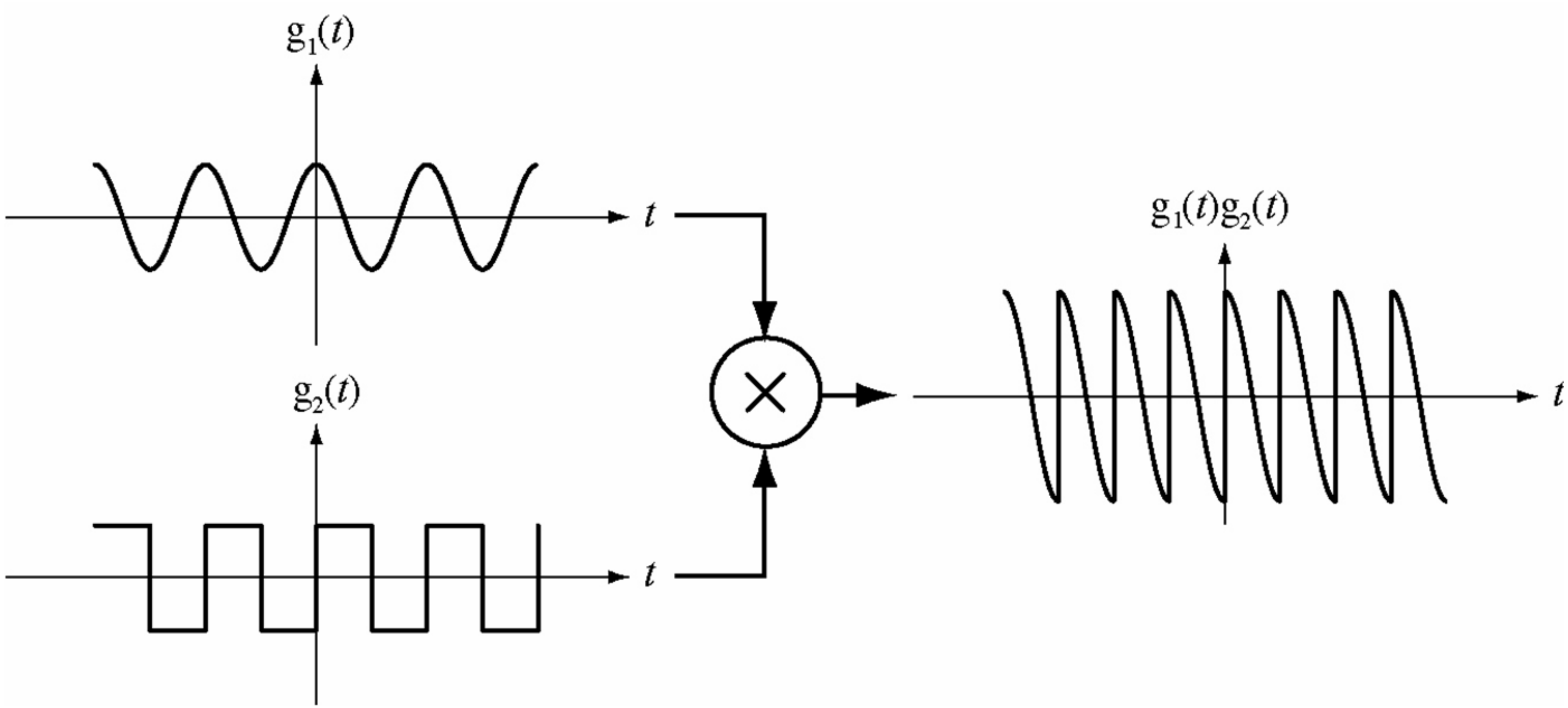
Decomposition of Even and Odd Functions

Product of Two Even Functions



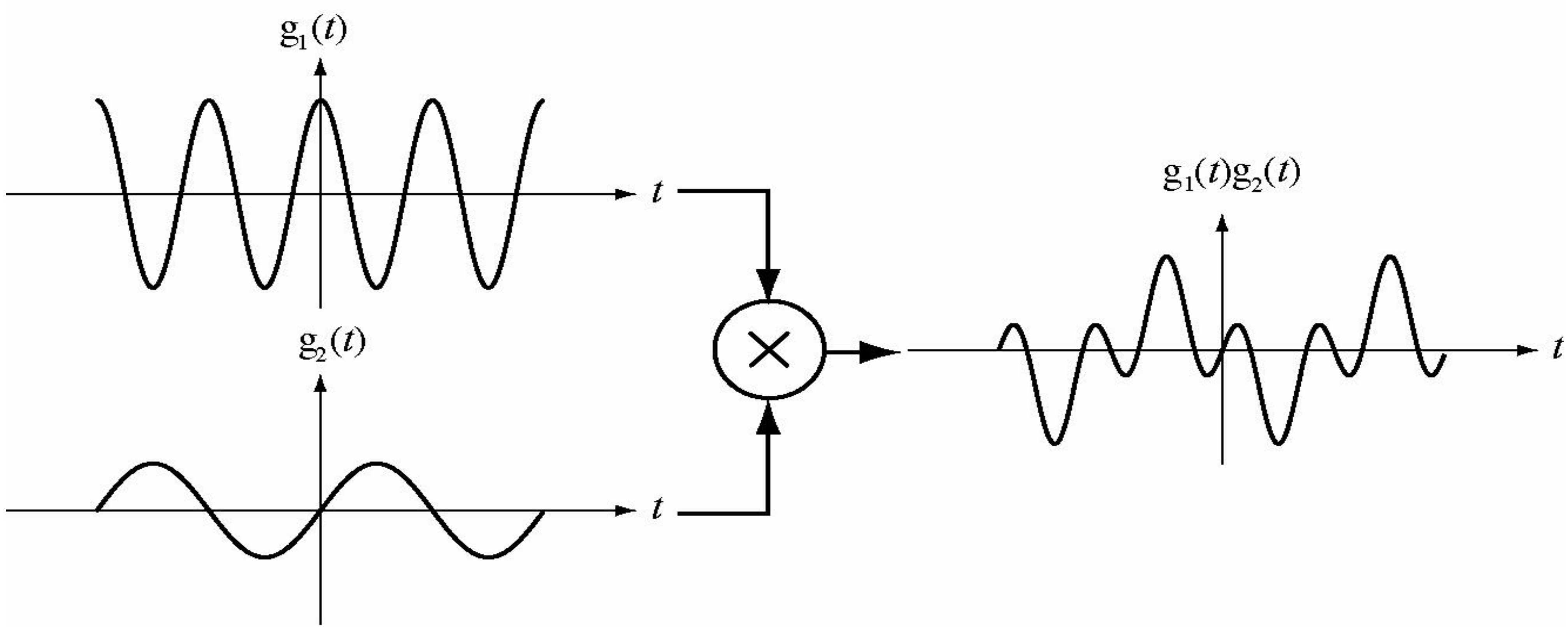
Decomposition of Even and Odd Functions

Product of an Even Function and an Odd Function



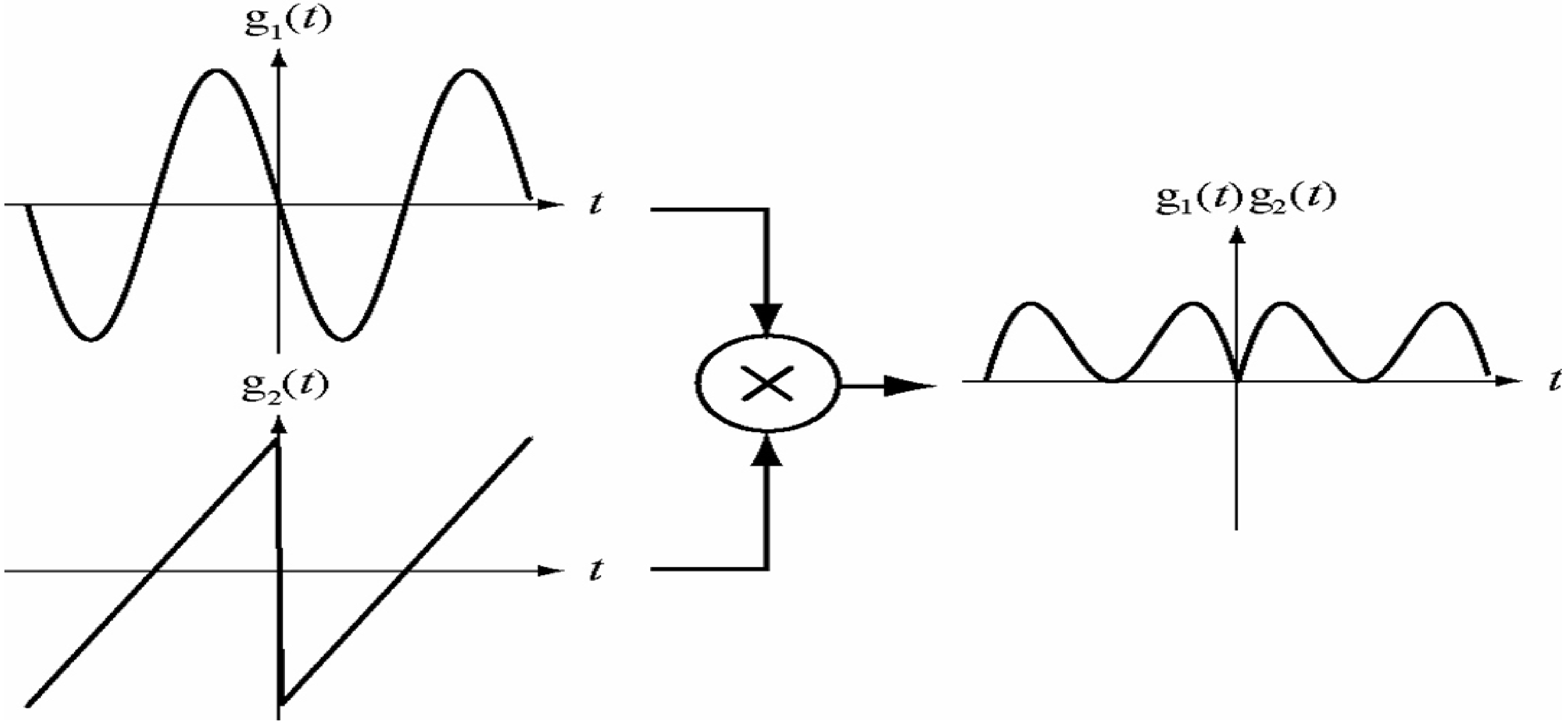
Decomposition of Even and Odd Functions

Product of an Even Function and an Odd Function



Decomposition of Even and Odd Functions

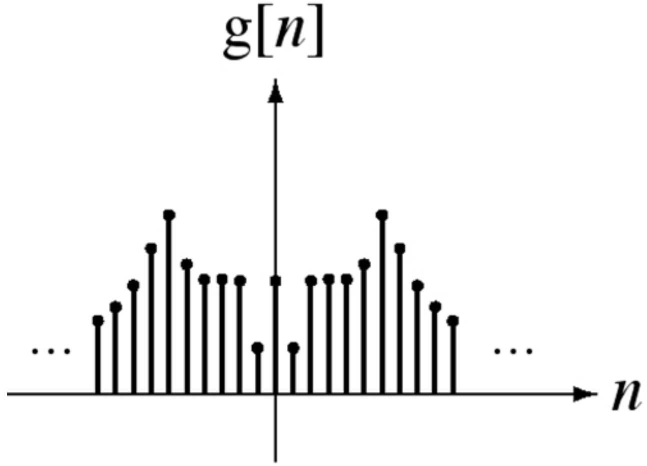
Product of Two Odd Functions



Discrete Time Even and Odd Signals

$$g[n] = g[-n]$$

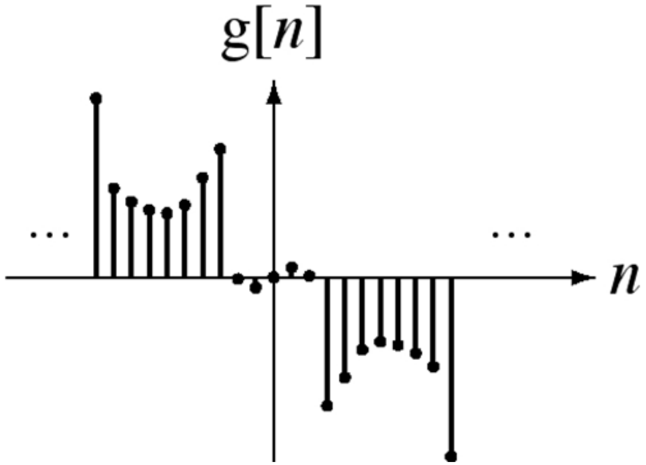
Even Function



$$g_e[n] = \frac{g[n] + g[-n]}{2}$$

$$g[n] = -g[-n]$$

Odd Function



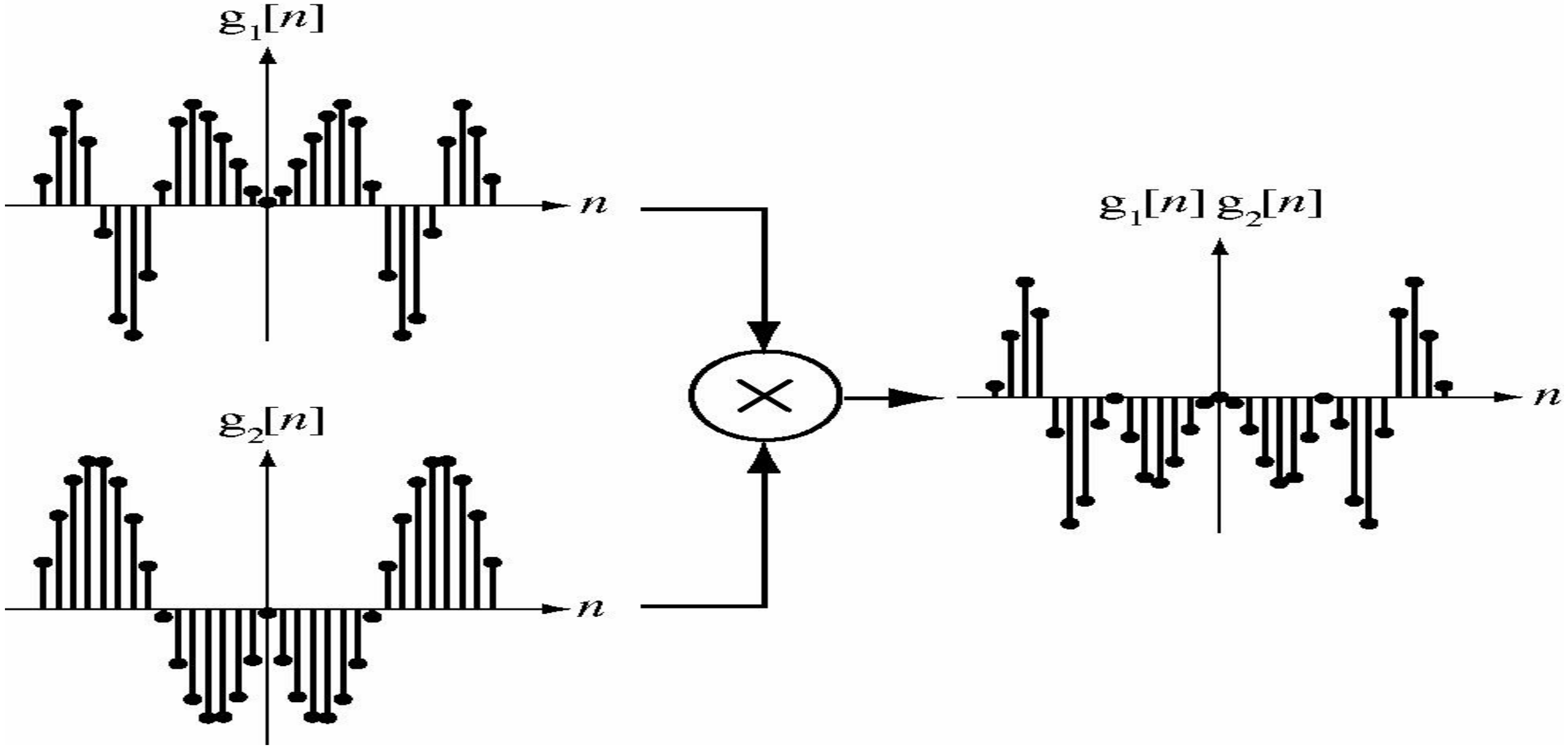
$$g_o[n] = \frac{g[n] - g[-n]}{2}$$

Combination of even and odd function for DT Signals

Function type	Sum	Difference	Product	Quotient
Both even	Even	Even	Even	Even
Both odd	Odd	Odd	Even	Even
Even and odd	Even or Odd	Even or odd	Odd	Odd

Decomposition of DT Even and Odd Functions

Two Even Functions



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Energy and Power Signals

Energy Signal

- A signal with finite energy and zero power is called Energy Signal i.e. for energy signal, $0<E<\infty$ and $P=0$
- Signal energy of a signal is defined as the area under the square of the magnitude of the signal.

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

Energy and Power Signals

Power Signal

- Some signals have infinite signal energy. In that case it is more convenient to deal with average signal power
- For power signals: $0<P<\infty$ and $E = \infty$
- Average power of the signal is given by

$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt$$

Signal Detection Theory

Can we be sure it was a signal?

How do we know it's the right signal?

How do we decide to act or not?

Signal Detection Theory

Signal detection theory is based on 3 assumptions:

- Noise
- Signal
- Decision

Signal Detection Theory

Signal detection theory is based on 3 assumptions:

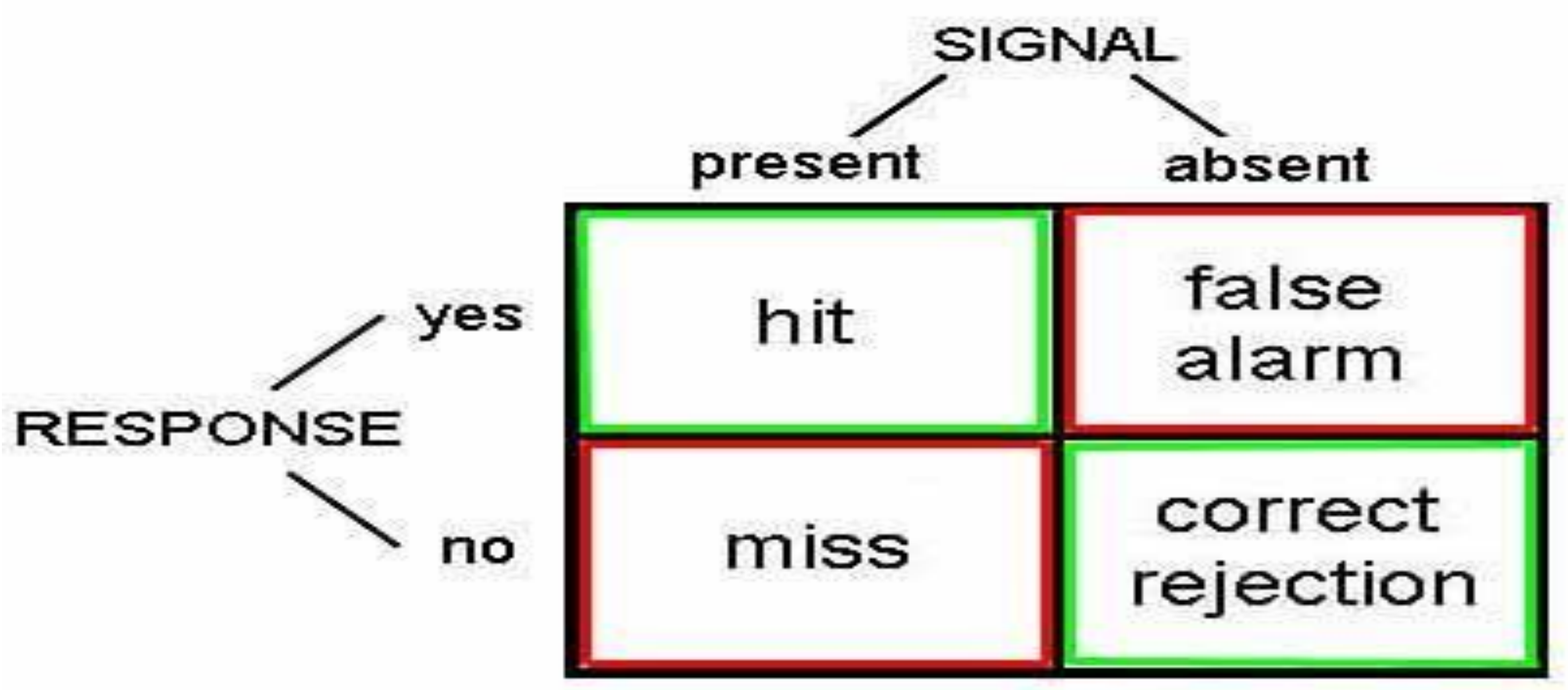
- While we are at the mercy of our senses and how good they are
- We also have to make decisions about what action to take
- Decisions are dependent upon a number of things

Signal Detection Theory

Judgment

The performer has to make a decision based upon a judgments of the situational variables, and interpretation (judge) of the sensory information available within each situation

Signal Detection Theory



Signal Detection Theory

- If a signal is present and a person correctly identifies the signal, then she has made a 'hit.' (top left)
- However, if the signal is absent and she says that the signal is present, then she has made a 'false alarm.’ (top left)
- If the signal present but she says it is not, she made a ‘miss.’ (bottom left)
- If the signal is absent, and she says it’s absent, she made a ‘correct rejection.’ (bottom right)

Signal Detection Theory

There are three variables to consider:

- Internal activity (‘ Noise ’ in the system)
- Input from senses (environment), we call the Signal, which can sometimes be confusing
- Cut-off point (accept or reject)

Signal Detection Theory

Internal Noise

- Anxiety / stress raises internal activity
- Uncertainty
- Lack of experience / indecision

Signal Detection Theory

External Noise

- Weak signal
- Confusion (competing signals)
- Uncertainty (is it or not)
- Lack of experience (looking wrong place, etc.)

Signal Detection Theory

Cut-off (accept / reject)

- Experience
- What’s at stake

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References

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3. Giron-Sierra, Jose Maria. Digital Signal Processing with Matlab Examples, Volume 1: Signals and Data, Filtering, Non-stationary Signals, Modulation. Springer, 2016.

THANK YOU